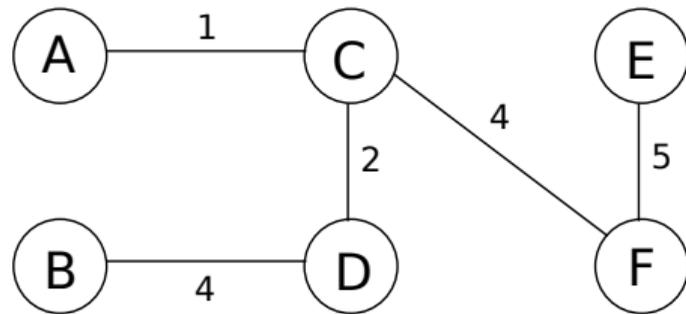
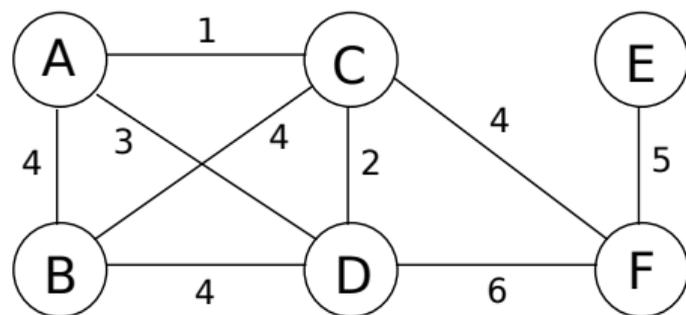


# Outline

Greedy  
MST

# Minimum spanning tree



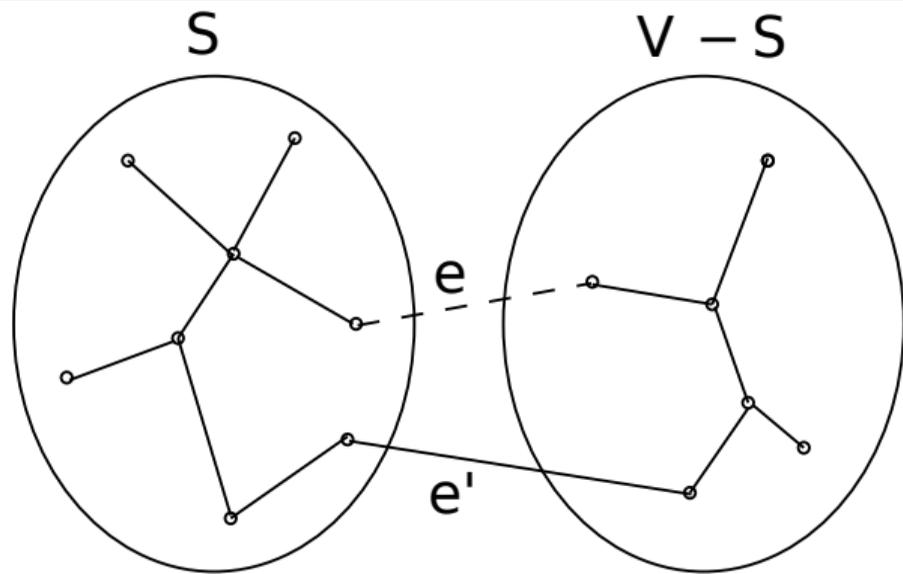
## Minimum Spanning Tree

Given  $G = (V, E)$ ,  $w : E \rightarrow \mathbb{R}$ , a *minimum spanning tree*  $T$  is a spanning tree (i.e. connecting all vertices) that minimizes  $\text{cost}(T) = \sum_{e \in T} w(e)$

# Cut Property

## Lemma

Let  $T$  be a minimum spanning tree,  $X \subset T$  s.t.  $X$  does not connect  $(S, V - S)$ . Let  $e$  be the lightest edge from  $S$  to  $V - S$ .  $X \cup e$  is part of some MST.



# Cut Property Proof

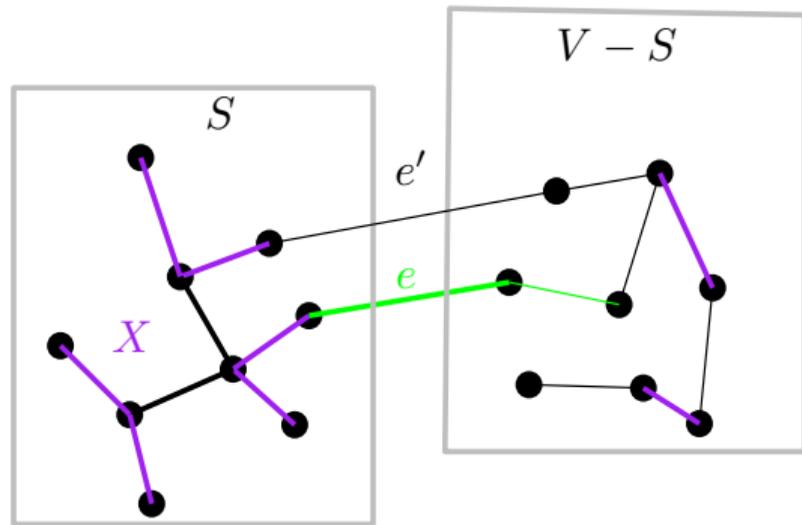
## Cut Property

Let  $T$  be an MST,  $X \subset T$  s.t.  $X$  does not connect  $(S, V - S)$ . Let  $e$  be the lightest edge from  $S$  to  $V - S$ .  $X \cup e$  is part of some MST.

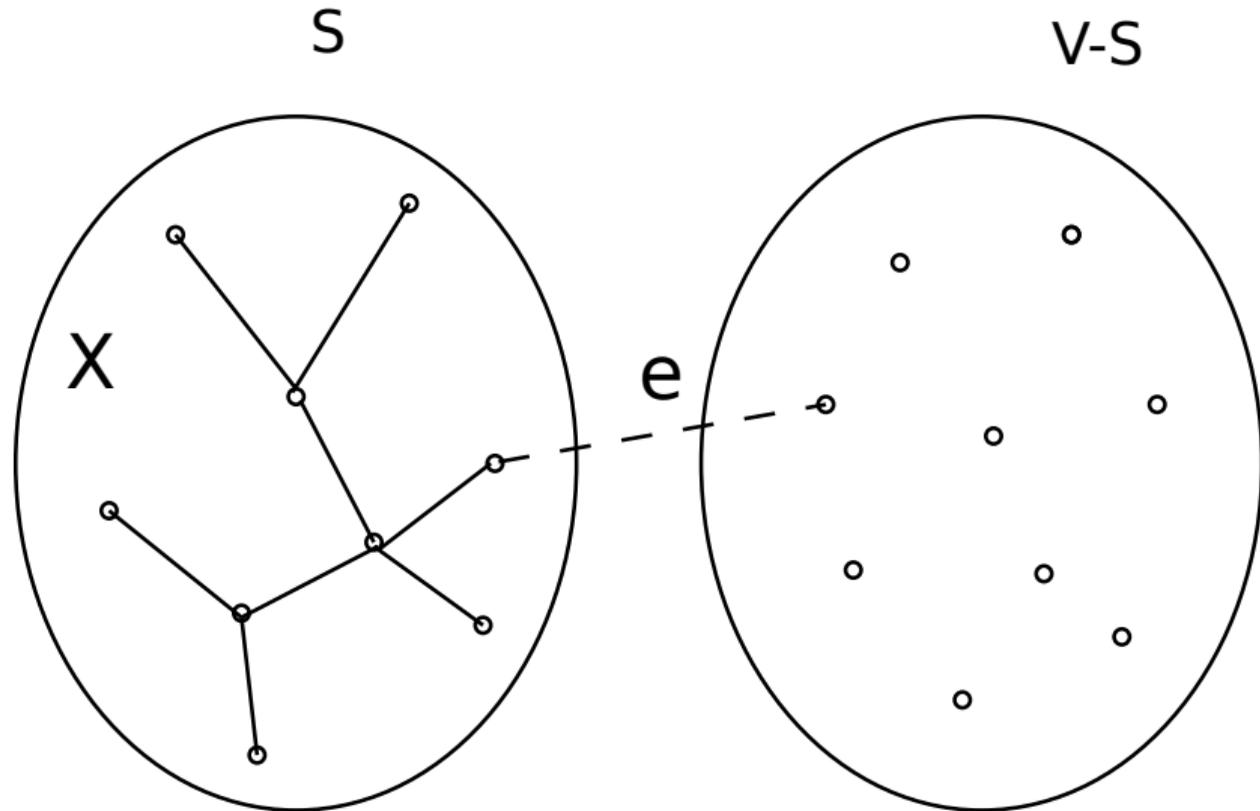
- ▶ Let  $X \subseteq T$  where  $T$  is MST
- ▶ if  $e \in T$ , done
- ▶ add  $e$  to  $T$ , makes a cycle

# Cut Property Proof

- ▶ Let  $X \subseteq T$  where  $T$  is MST
- ▶ if  $e \in T$ , done
- ▶ add  $e$  to  $T$ , makes a cycle
- ▶  $\exists$  crossing  $e' \in E - X$
- ▶ swap  $e$  and  $e'$



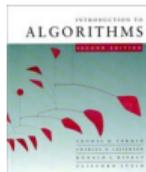
# Prim's Algorithm



$S$  = nodes reached so far

# Prim's Algorithm

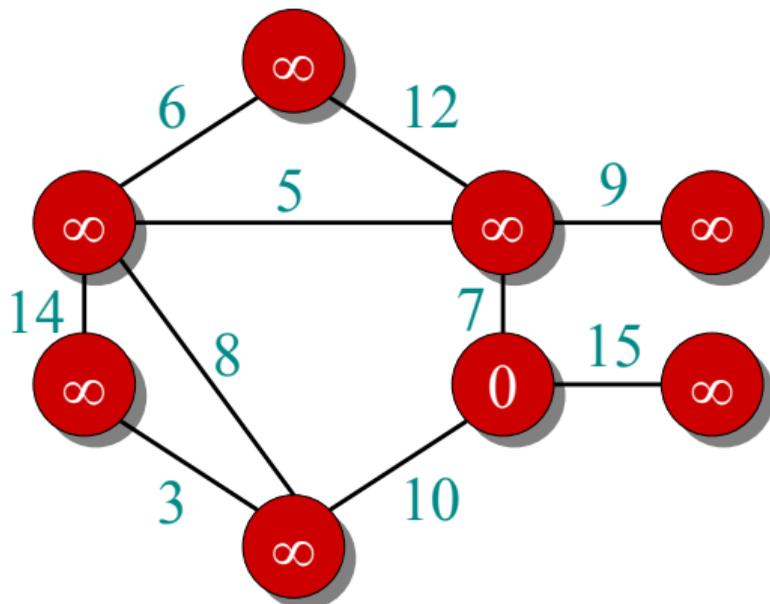
```
def prim(G,root):
    pq = pqdict(); prev = {}
    for v in G.keys():
        pq.additem(v,inf)
    pq.updateitem(root,0)
    while len(pq)>0:
        v = pq.pop()
        for (z,weight) in G[v]:
            if z in pq and weight < pq[z]:
                prev[z]=v
                pq.updateitem(z,weight)
    return prev
```

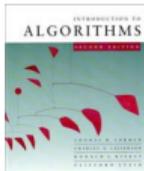


# Example of Prim's algorithm

○  $\in A$

●  $\in V - A$

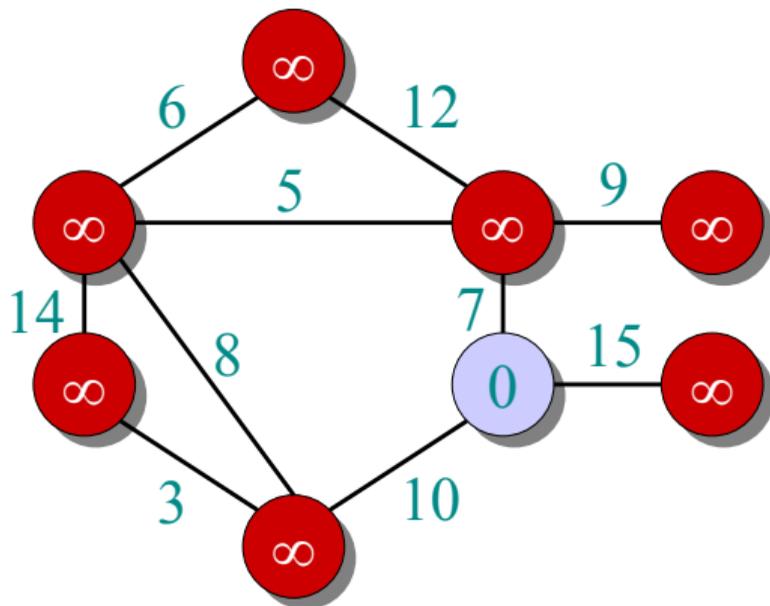


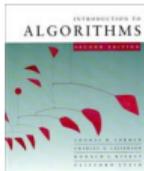


# Example of Prim's algorithm

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●  $\in V - A$

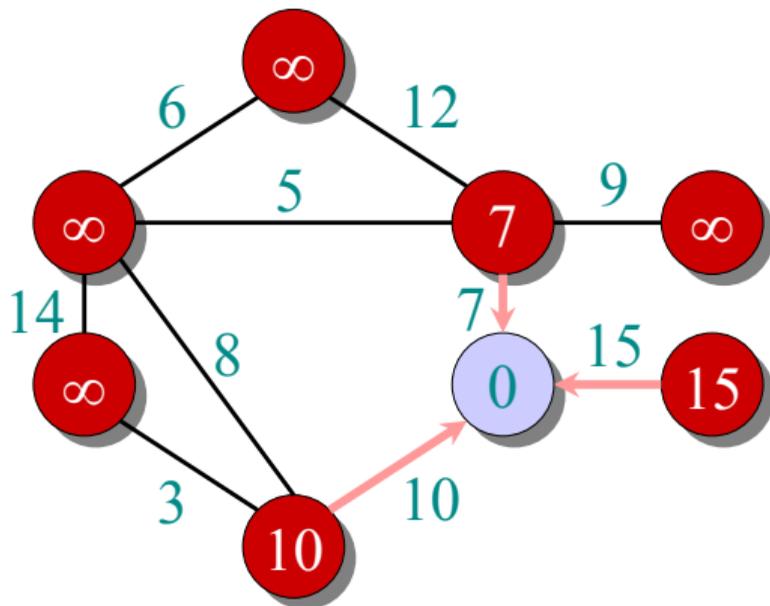


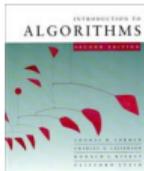


# Example of Prim's algorithm

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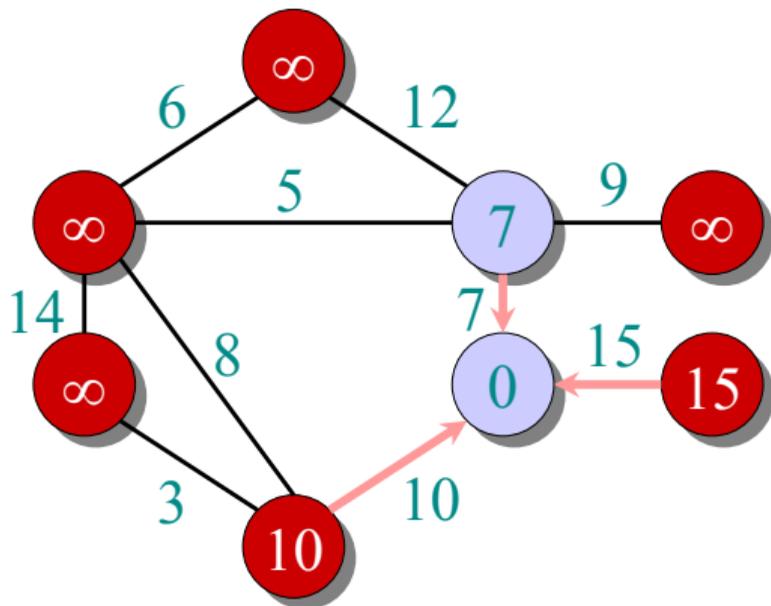


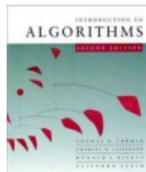


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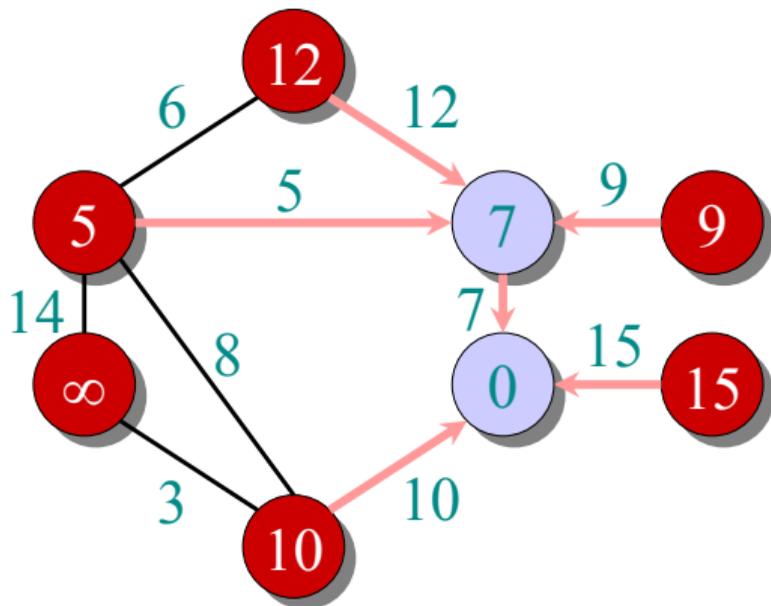


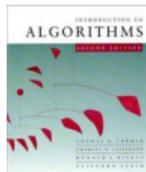


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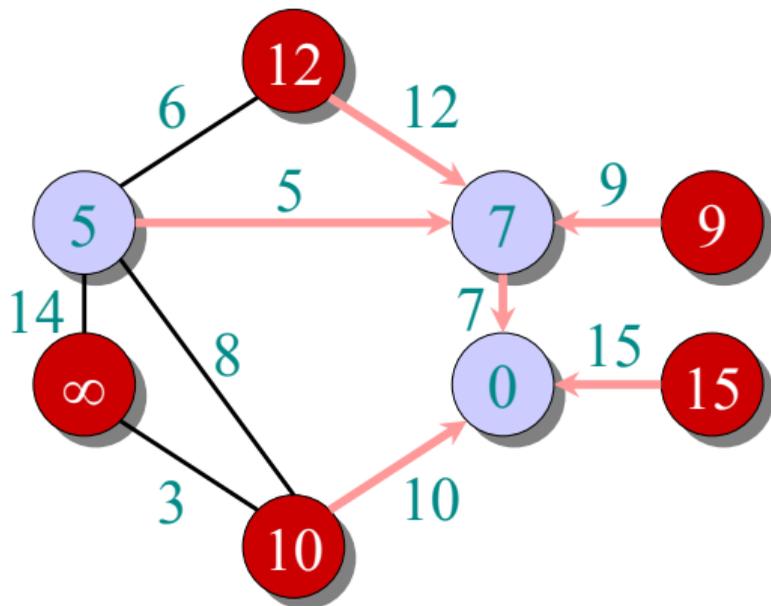


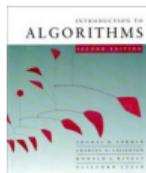


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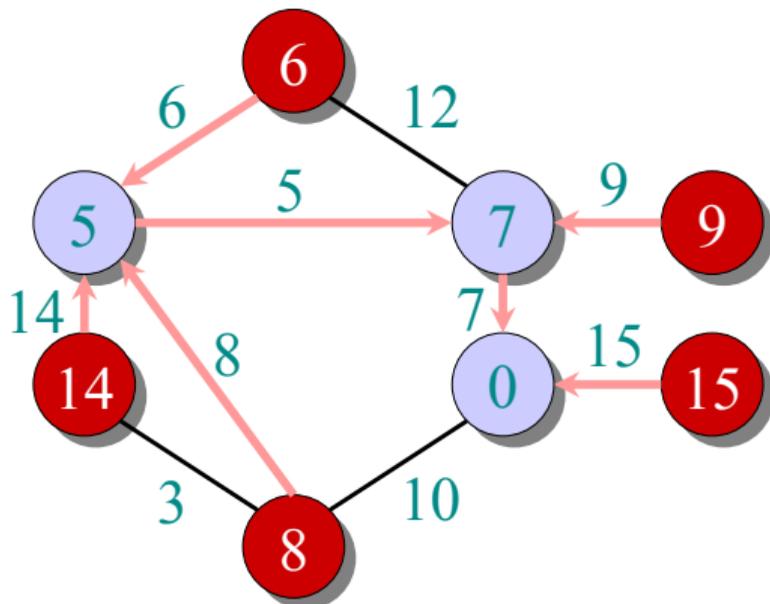


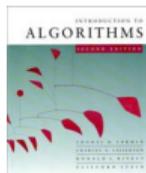


# Example of Prim's algorithm

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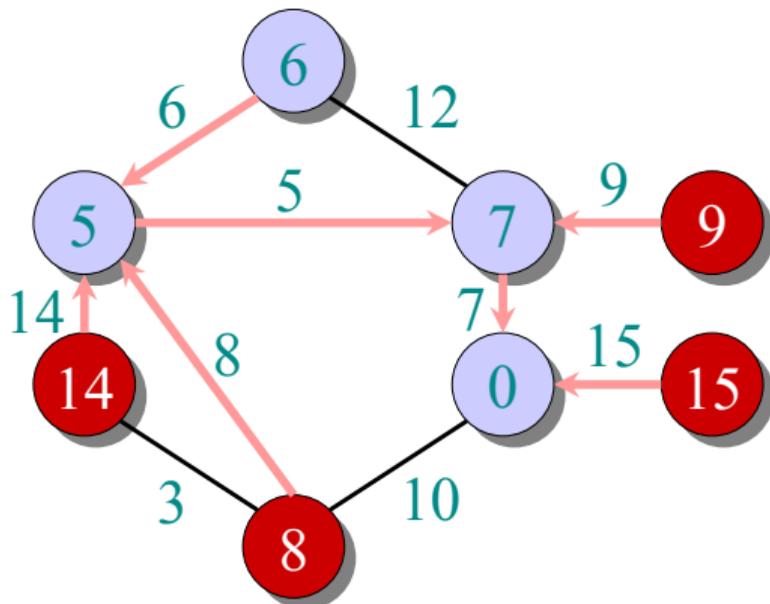


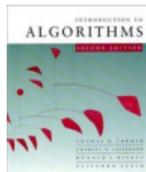


# Example of Prim's algorithm

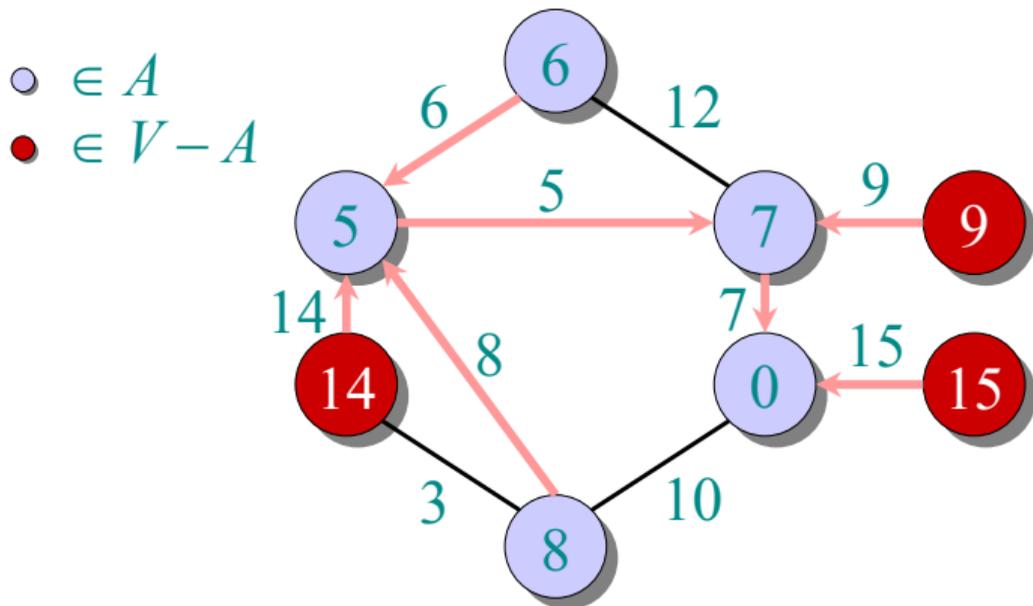
○  $\in A$

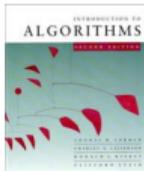
●  $\in V - A$





# Example of Prim's algorithm

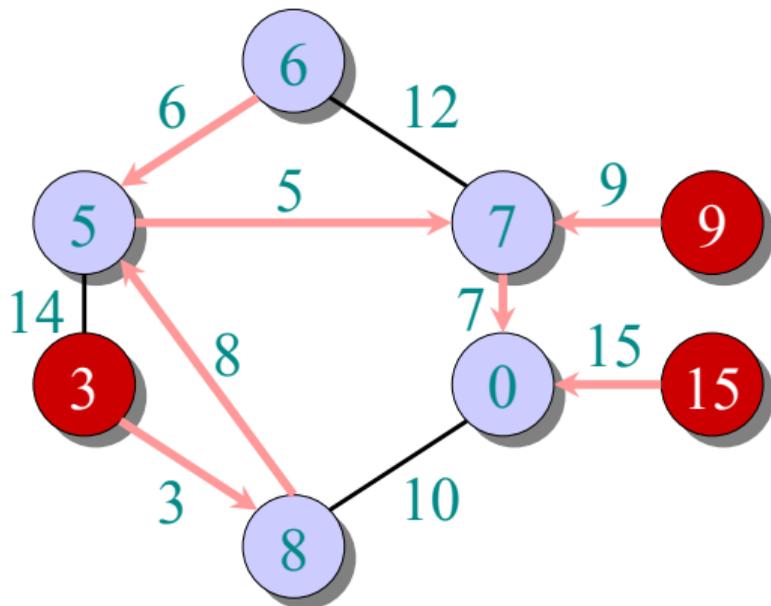


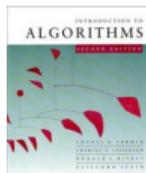


# Example of Prim's algorithm

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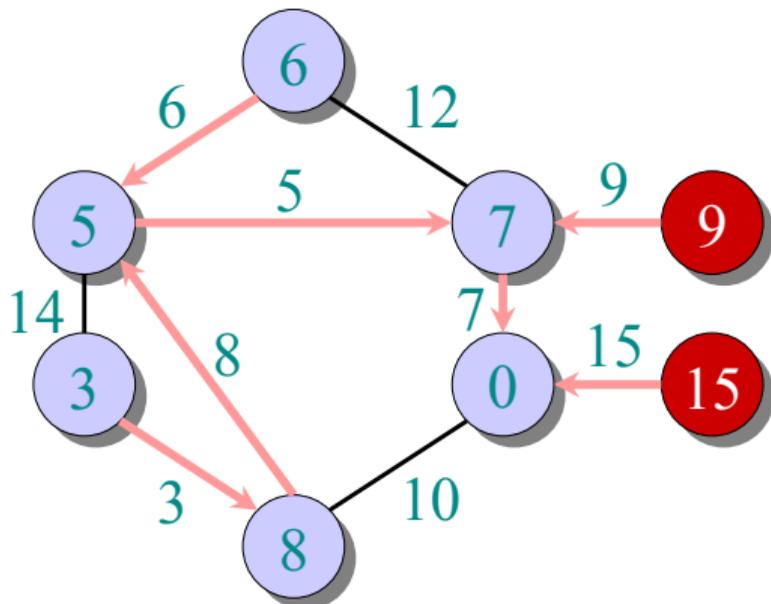


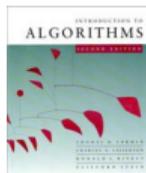


# Example of Prim's algorithm

○  $\in A$

●  $\in V - A$





# Example of Prim's algorithm

○  $\in A$

●  $\in V - A$

