CS3383 Unit 0: Asymptotics Review

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Unit prereqs

- O and Ω (CS2383)
- limits, derivatives (calculus)
- induction (CS1303)
- working with inequalities
- monotone functions

The Big Question(s)

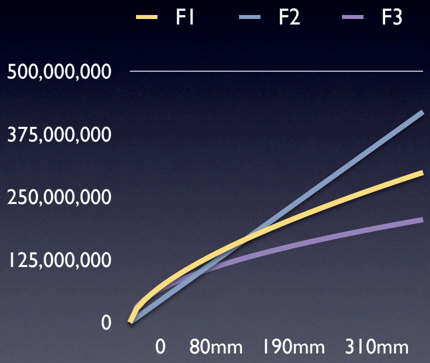
- When is Algorithm A better than Algorithm B w.r.t. running time and memory use?
- If we know the input, we can just run the two algorithms.
- In general we assume performance is a function of the input size (bits / bytes)
- So we need to know how to compare functions.
- We also need not to drown in details.

Asymptotic Notation

f1: $1{,}000 * n^{0.635}$

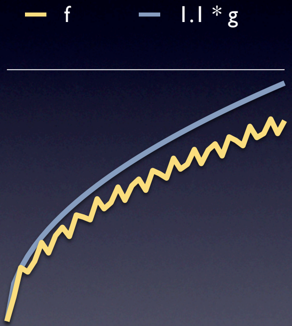
f2: n

f3: $10{,}000 * n^{0.5}$



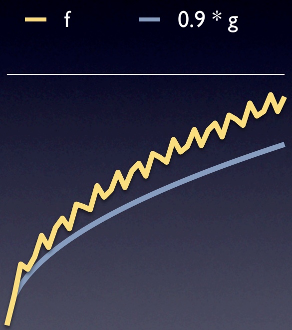
Asymptotic Notation

- f = O(g)

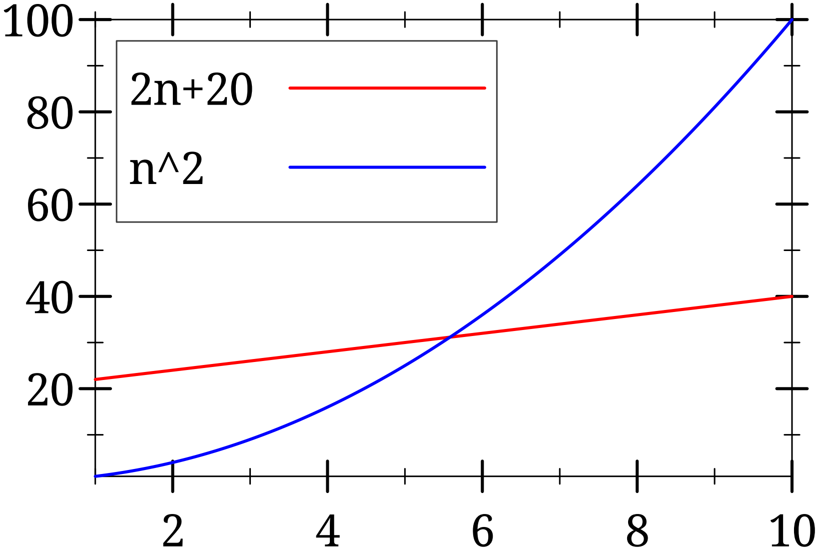


Asymptotic Notation

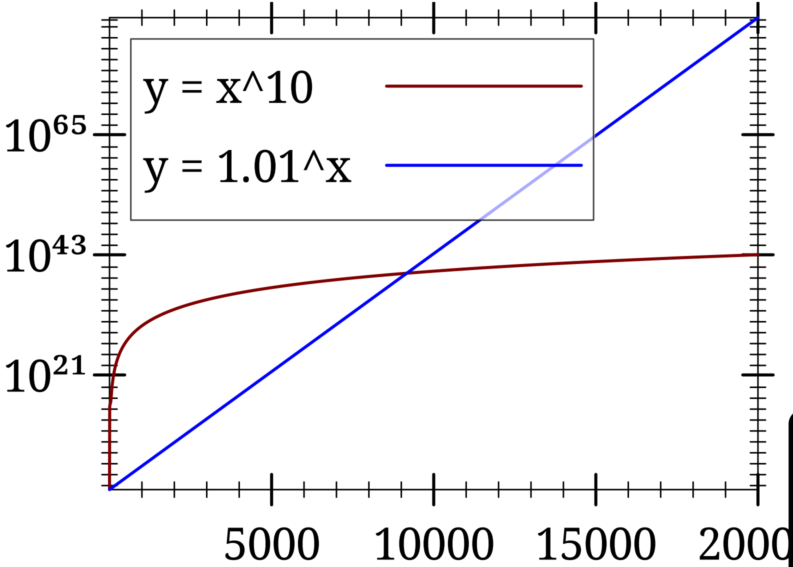
- $f = \Omega(g)$

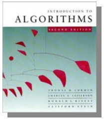


Linear versus Quadratic



Exponential versus Polynomial

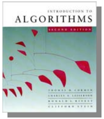




Asymptotic notation

O-notation (upper bounds):

> We write $f(n) = O(g(n))$ if there exist constants $c > 0$, $n_0 > 0$ such that $0 \le f(n) \le cg(n)$ for all $n \ge n_0$.



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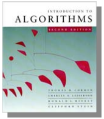
EXAMPLE: $2n^2 = O(n^3)$ $\quad (c = 1, n_0 = 2)$



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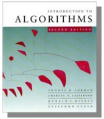
functions, not values



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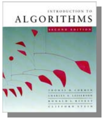
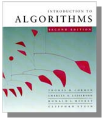
functions, not values

funny, "one-way" equality



Set definition of O-notation

$O(g(n)) = \{ f(n) :$ there exist constants $c > 0,\ n_0 > 0$ such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0 \}$



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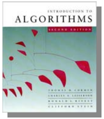
EXAMPLE: $2n^2 \in O(n^3)$



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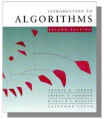
EXAMPLE: $2n^2 \in O(n^3)$

(*Logicians:* $\lambda n.2n^2 \in O(\lambda n.n^3)$, but it's convenient to be sloppy, as long as we understand what's *really* going on.)

Macro substitution

Convention: A set in a formula represents an anonymous function in the set.



Macro substitution

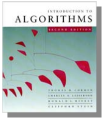
Convention: A set in a formula represents an anonymous function in the set.

EXAMPLE: $f(n) = n^3 + O(n^2)$

means

$f(n) = n^3 + h(n)$

for some $h(n) \in O(n^2)$.



Macro substitution

Convention: A set in a formula represents an anonymous function in the set.

EXAMPLE: $n^2 + O(n) = O(n^2)$

means

for any $f(n) \in O(n)$:

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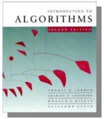
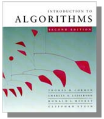


Ω-notation (lower bounds)

O-notation is an *upper-bound* notation. It makes no sense to say $f(n)$ is at least $O(n^2)$.



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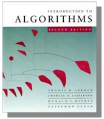

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EXAMPLE: $\sqrt{n} = \Omega(\lg n) \quad (c = 1,\ n_0 = 16)$